International Journal of Advanced Research in Engineering Technology & Sciences ISSN: 2394-2819
July-2016 Volume 3, Issue-7
Email:editor@ijarets.org www.ijarets.org

HOMOMORPHISM ON FUZZY HX Ring

R. Muthuraj
Assistant Professor
Department of Mathematics
H.H.The Rajah's College
Pudukkottai

N.Ramila Gandhi
Associate Professor
Department of Mathematics
PSNA College of Engineering and Technology
Dindigul

ABSTRACT: *In this paper, we introduce the concept of an image, pre-image of fuzzy subset of a HX ring and discuss the properties of homomorphism and anti homomorphism images and pre -images of fuzzy HX subring of a HX ring.*

KEYWORDS: *HX ring, fuzzy HX ring, homomorphism and anti homomorphism of fuzzy HX ring, Image and pre-image of fuzzy sets.*

1. INTRODUCTION

In 1965, Zadeh [12] introduced the concept of fuzzy subset of a set X as a function from X into the closed interval 0 and 1 and studied their properties. Fuzzy set theory is a useful tool to describe situations in which the data or imprecise or vague and it is applied to logic , set theory, group theory, ring theory, real analysis, measure theory etc. In 1967, Rosenfeld [8] defined the idea of fuzzy subgroups and gave some of its properties. Li Hong Xing [3] introduced the concept of HX group. In 1988, Professor Li Hong Xing [5] proposed the concept of HX ring and derived some of its properties, then Professor Zhong [1,2] gave the structures of HX ring on a class of ring. In this paper we define a new algebraic structure of an fuzzy HX subring of a HX ring and investigate some related properties**.**

2. PRELIMINARIES

In this section, we site the fundamental definitions that will be used in the sequel. Throughout this paper, R = (R ,+, ·) is a ring, e is the additive identity element of R and xy, we mean x.y.

2.1 Definition

Let R be a ring. In $2^R - \{\phi\}$, a non-empty set $\vartheta \subset 2^R - \{\phi\}$ with two binary operations ' + ' and '·' is said to be a HX ring on R if ϑ is a ring with respect to the algebraic operation defined by

i. A + B = {a + b / a $\in$ A and b $\in$ B} , which its null element is denoted by Q , and the negative element of A is denoted by – A.

ii. AB = {ab / a $\in$ A and b $\in$ B}

iii. A (B + C) = AB + AC and (B + C) A = BA + CA.

2.2 Definition

Let R be a ring. Let μ be a fuzzy ring defined on R . Let $\vartheta \subset 2^R - \{\phi\}$ be a HX ring. A fuzzy subset λ^μ of ϑ is called a fuzzy HX ring on ϑ or a fuzzy ring induced by μ if the following conditions are satisfied. For all A , B $\in \vartheta$,

i. λ^μ (A – B) $\geq$ min { λ^μ (A), λ^μ (B) } ,

ii. λ^μ (AB) $\geq$ min { λ^μ (A), λ^μ (B) }

where $\lambda^\mu(A) = \max \{ \mu(x) \ / \ \text{for all } x \in A \subseteq R \}$.

3. IMAGE AND PRE-IMAGE OF A FUZZY HX RING OF A HX RING UNDER HOMOMORPHISM AND ANTI HOMOMORPHISM

In this section, we introduce the concept of an image, pre-image of the fuzzy sub HX ring of a HX ring and discuss some of its properties.

3.1 *Definition*

Let R_1 and R_2 be any two rings. Let $\Re_1 \subset 2^{R1} - \{\phi\}$ and $\Re_2 \subset 2^{R2} - \{\phi\}$ be any two HX rings defined on R_1 and R_2 respectively. Let μ and α be any two fuzzy subsets in R_1 and R_2 respectively. Let λ^{μ} and η^{α} be fuzzy subsets defined on $\Re_1$ and $\Re_2$ respectively induced by μ and α. Let f: $\Re_1 \rightarrow \Re_2$ be a mapping then the image of λ^{μ} denoted as f (λ^{μ}) is a fuzzy subset of $\Re_2$ defined as for each $U \in \Re_2$,

$$(f(\lambda^{\mu}))(U) = \begin{cases} \sup\{\lambda^{\mu}(X) : X \in f^{-1}(U)\}, & \text{if } f^{-1}(U) \neq \phi \\ 0, & \text{otherwise} \end{cases}$$

Also the pre-image of η^{α} denoted as $f^{-1}(\eta^{\alpha})$ under f is a fuzzy subset of $\Re_1$ defined as for each $X \in \Re_1$, $(f^{-1}(\eta^{\alpha}))(X) = \eta^{\alpha}(f(X))$.

3.2 Definition

Let R_1 and R_2 be any two rings. Let $\Re_1 \subset 2^{R1} - \{\phi\}$ and $\Re_2 \subset 2^{R2} - \{\phi\}$ be any two HX rings defined on R_1 and R_2 respectively. Let μ and α be any two fuzzy subsets in R_1 and R_2 respectively. Let λ^{μ} and η^{α} be fuzzy subsets defined on $\Re_1$ and $\Re_2$ respectively induced by μ and α. The mapping f: $\Re_1 \rightarrow \Re_2$ is said to be a homomorphism if it satisfies the following conditions. For any A, B $\in \Re_1$,

i. $f(A+B) = f(A) + f(B)$
ii. $f(AB) = f(A) \cdot f(B)$.

3.3 Definition

Let R_1 and R_2 be any two rings. Let $\Re_1 \subset 2^{R1} - \{\phi\}$ and $\Re_2 \subset 2^{R2} - \{\phi\}$ be any two HX rings defined on R_1 and R_2 respectively. Let μ and α be any two fuzzy subsets in R_1 and R_2 respectively. Let λ^{μ} and η^{α} be fuzzy subsets defined on $\Re_1$ and $\Re_2$ respectively induced by μ and α. The mapping f: $\Re_1 \rightarrow \Re_2$ is said to be an anti homomorphism if it satisfies the following conditions. For any A, B $\in \Re_1$,

i. $f(A+B) = f(A) + f(B)$
ii. $f(AB) = f(B) \cdot f(A)$.

3.4 Theorem

Let $\Re_1$ and $\Re_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let f : $\Re_1 \rightarrow \Re_2$ be a homomorphism onto HX rings. Let λ^{μ} be a fuzzy HX subring of $\Re_1$ then f (λ^{μ}) is a fuzzy HX subring of $\Re_2$, if λ^{μ} has a supremum property and λ^{μ} is f-invariant.

Proof

Let μ be a fuzzy subset of R_1 and λ^{μ} is a fuzzy HX subring of $\Re_1$.

There exist $X, Y \in \Re_1$ such that $f(X), f(Y) \in \Re_2$,

$$\begin{aligned} (f(\lambda^{\mu}))(f(X)-f(Y)) &= (f(\lambda^{\mu}))(f(X-Y)), \\ &= \lambda^{\mu}(X-Y) \\ &\geq \min\{\lambda^{\mu}(X), \lambda^{\mu}(Y)\} \\ &= \min\{(f(\lambda^{\mu}))(f(X)), (f(\lambda^{\mu}))(f(Y))\} \\ (f(\lambda^{\mu}))(f(X)-f(Y)) &\geq \min\{(f(\lambda^{\mu}))(f(X)), (f(\lambda^{\mu}))(f(Y))\}. \end{aligned}$$

i. $$\begin{aligned} f(\lambda^{\mu}))(f(X)\,f(Y)) &= (f(\lambda^{\mu}))(f(XY)), \\ &= \lambda^{\mu}(XY) \\ &\geq \min\{\lambda^{\mu}(X), \lambda^{\mu}(Y)\} \\ &= \min\{(f(\lambda^{\mu}))(f(X)), (f(\lambda^{\mu}))(f(Y))\} \\ (f(\lambda^{\mu}))(f(X)f(Y)) &\geq \min\{(f(\lambda^{\mu}))(f(X)), (f(\lambda^{\mu}))(f(Y))\}. \end{aligned}$$

Hence, $f(\lambda^{\mu})$ is a fuzzy HX subring of $\Re_2$.

3.5 Theorem

Let $\Re_1$ and $\Re_2$ be any two HX rings on R_1 and R_2 respectively. Let f: $\Re_1 \rightarrow \Re_2$ be a homomorphism on HX rings. Let η^{α} be a fuzzy HX subring of $\Re_2$ then $f^{-1}(\eta^{\alpha})$ is a fuzzy HX subring of $\Re_1$.

Proof

Let α be a fuzzy subset of R_2 and η^{α} be a fuzzy HX subring of $\Re_2$.

For any $X, Y \in \Re_1$, $f(X), f(Y) \in \Re_2$,

i. $(f^{-1}(\eta^{\alpha}))(X-Y) = \eta^{\alpha}(f(X-Y))$
$= \eta^{\alpha}(f(X) - f(Y))$,
$\geq \min\{\eta^{\alpha}(f(X)), \eta^{\alpha}(f(Y))\}$
$= \min\{(f^{-1}(\eta^{\alpha}))(X), (f^{-1}(\eta^{\alpha}))(Y)\}$
$(f^{-1}(\eta^{\alpha}))(X-Y) \geq \min\{(f^{-1}(\eta^{\alpha}))(X), (f^{-1}(\eta^{\alpha}))(Y)\}$.

ii. $(f^{-1}(\eta^{\alpha}))(XY) = \eta^{\alpha}(f(XY))$
$= \eta^{\alpha}(f(X)f(Y))$
$\geq \min\{\eta^{\alpha}(f(X)), \eta^{\alpha}(f(Y))\}$
$= \min\{(f^{-1}(\eta^{\alpha}))(X), (f^{-1}(\eta^{\alpha}))(Y)\}$
$(f^{-1}(\eta^{\alpha}))(XY) \geq \min\{(f^{-1}(\eta^{\alpha}))(X), (f^{-1}(\eta^{\alpha}))(Y)\}$.

Hence, $f^{-1}(\eta^{\alpha})$ is a fuzzy HX subring of $\Re_1$.

3.6 Theorem

Let $\Re_1$ and $\Re_2$ be any two HX rings on R_1 and R_2 respectively. Let f: $\Re_1 \rightarrow \Re_2$ be an anti homomorphism onto HX rings. Let λ^{μ} be a fuzzy HX subring of $\Re_1$, then $f(\lambda^{\mu})$ is a fuzzy HX subring of $\Re_2$, if λ^{μ} has a supremum property and λ^{μ} is f-invariant.

Proof

Let μ be a fuzzy subset of R_1 and λ^{μ} is a fuzzy HX subring of $\Re_1$, then

There exist $X, Y \in \Re_1$ such that $f(X), f(Y) \in \Re_2$

i. $(f(\lambda^{\mu}))(f(X) - f(Y)) = (f(\lambda^{\mu}))(f(Y-X))$,
$= \lambda^{\mu}(Y-X)$
$\geq \min\{\lambda^{\mu}(Y), \lambda^{\mu}(X)\}$
$= \min\{(f(\lambda^{\mu}))(f(Y)), (f(\lambda^{\mu}))(f(X))\}$
$(f(\lambda^{\mu}))(f(X) - f(Y)) \geq \min\{(f(\lambda^{\mu}))(f(X)), (f(\lambda^{\mu}))(f(Y))\}$.

ii. $(f(\lambda^{\mu}))(f(X)f(Y)) = (f(\lambda^{\mu}))(f(YX))$,
$= \lambda^{\mu}(YX)$
$\geq \min\{\lambda^{\mu}(Y), \lambda^{\mu}(X)\}$
$= \min\{(f(\lambda^{\mu}))(f(Y)), (f(\lambda^{\mu}))(f(X))\}$
$(f(\lambda^{\mu}))(f(X)f(Y)) \geq \min\{(f(\lambda^{\mu}))(f(X)), (f(\lambda^{\mu}))(f(Y))\}$.

Hence, $f(\lambda^{\mu})$ is a fuzzy HX subring of $\Re_2$.

3.7 Theorem

Let $\Re_1$ and $\Re_2$ be any two HX rings on R_1 and R_2 respectively. Let f: $\Re_1 \rightarrow \Re_2$ be an anti homomorphism on HX rings. Let η^{α} be a fuzzy HX subring of $\Re_2$ then $f^{-1}(\eta^{\alpha})$ is a fuzzy HX subring of $\Re_1$.

Proof

Let α be a fuzzy subset of R_2 and η^{α} be a fuzzy HX subring of $\Re_2$.

For any $X, Y \in \Re_1$, then $f(X), f(Y) \in \Re_2$

i. $(f^{-1}(\eta^{\alpha}))(X-Y) = \eta^{\alpha}(f(X-Y))$
$= \eta^{\alpha}(f(X) - f(Y))$,
$= \min\{(f^{-1}(\eta^{\alpha}))(X), (f^{-1}(\eta^{\alpha}))(Y)\}$
$(f^{-1}(\eta^{\alpha}))(X-Y) \geq \min\{(f^{-1}(\eta^{\alpha}))(X), (f^{-1}(\eta^{\alpha}))(Y)\}$.

ii. $(f^{-1}(\eta^{\alpha}))(XY) = \eta^{\alpha}(f(YX))$
$= \eta^{\alpha}(f(Y)\, f(X))$
$\geq \min\{\eta^{\alpha}(f(Y)), \eta^{\alpha}(f(X))\}$
$= \min\{(f^{-1}(\eta^{\alpha}))(Y), (f^{-1}(\eta^{\alpha}))(X)\}$
$(f^{-1}(\eta^{\alpha}))(XY) \geq \min\{(f^{-1}(\eta^{\alpha}))(Y), (f^{-1}(\eta^{\alpha}))(X)\}$.

Therefore, $f^{-1}(\eta^{\alpha})$ is a fuzzy HX subring of $\Re_1$.

4. LEVEL SUBSETS OF FUZZY HX RING

In this section, we introduce the idea of a level subsets of a fuzzy HX ring. We also discuss the relation between a given fuzzy HX subring of a HX ring and its level sub HX rings and investigate the conditions under which a given HX ring has a properly inclusive chain of sub HX rings.

4.1 Definition

Let λ^{μ} be a fuzzy HX subring of a HX ring $\Re$. For any $t \in [0,1]$, we define the set $U(\lambda^{\mu}; t) = \{ A \in \Re / \lambda^{\mu}(A) \geq t \}$ is called an upper level subset or a level subset of λ^{μ}.

4.2 Theorem

Let λ^{μ} be a fuzzy HX subring of a HX ring $\Re$ and $U(\lambda^{\mu}; t)$ is non-empty, then for any $t \in [0,1]$, $U(\lambda^{\mu}; t)$ is a sub HX ring of $\Re$.

Proof

Let λ^{μ} be a fuzzy HX subring of a HX ring $\Re$.

For any $A, B \in U(\lambda^{\mu}; t)$ we have, $\lambda^{\mu}(A) \geq t$ and $\lambda^{\mu}(B) \geq t$.

Now, $\lambda^{\mu}(A - B) \geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$
$\geq \min\{t, t\} = t$, for some $t \in [0,1]$
$\lambda^{\mu}(A - B) \geq t$
$\lambda^{\mu}(AB) \geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$
$\geq \min\{t, t\} = t$
$\lambda^{\mu}(AB) \geq t$

Hence, $A - B$, $AB \in U(\lambda^{\mu}; t)$.

Hence, $U(\lambda^{\mu}; t)$ is a sub HX ring of a HX ring $\Re$.

4.3 Theorem

Let $\Re$ be a HX ring and λ^{μ} be a fuzzy subset of $\Re$ such that $U(\lambda^{\mu}; t)$ is a sub HX ring of $\Re$ for all $t \in [0,1]$ then λ^{μ} is a fuzzy HX subring of $\Re$.

Proof

Let $A, B \in \Re$,
Let $A \in U(\lambda^{\mu}; t_1) \Rightarrow \lambda^{\mu}(A) \geq t_1$
and $B \in U(\lambda^{\mu}; t_2) \Rightarrow \lambda^{\mu}(B) \geq t_2$
Suppose $U(\lambda^{\mu}; t_1), U(\lambda^{\mu}; t_2) \in \Re$ and $A, B \in U(\lambda^{\mu}; t_2)$,
As $U(\lambda^{\mu}; t_2)$ is a sub HX ring of $\Re$,
$\lambda^{\mu}(A - B) \geq t_2$
$= \min\{t_1, t_2\}$

$$= \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$$
$$\lambda^{\mu}(A - B) \geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}.$$
$$\lambda^{\mu}(A B) \geq t_2$$
$$= \min\{t_1, t_2\}$$
$$= \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}.$$
$$\lambda^{\mu}(A B) \geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}.$$

Hence λ^{μ} is a fuzzy HX subring of $\mathfrak{R}$.

4.4 Theorem

A fuzzy subset λ^{μ} of $\mathfrak{R}$ is a fuzzy HX subring of a HX ring $\mathfrak{R}$ if and only if the level subsets $U(\lambda^{\mu}; t)$, $t \in$ Image λ^{μ}, are HX subrings of $\mathfrak{R}$.

Proof

It is clear.

4.5 Theorem

Let λ^{μ} be a fuzzy HX subring of a HX ring $\mathfrak{R}$. If two level sub HX rings, $U(\lambda^{\mu}; t_1)$, $U(\lambda^{\mu}; t_2)$ with $t_1 < t_2$ of λ^{μ} are equal if and only if there is no A in $\mathfrak{R}$ such that $t_1 \leq \lambda^{\mu}(A) < t_2$.

Proof

It is clear.

4.6 Theorem

Any sub HX ring H of a HX ring $\mathfrak{R}$ can be realized as a level sub HX ring of some fuzzy HX subring of $\mathfrak{R}$.

Proof

Let λ^{μ} be a fuzzy subset and $A \in \mathfrak{R}$.

$$\text{Define, } \lambda^{\mu}(A) = \begin{cases} t & \text{if } A \in H, \text{ where } t \in (0,1] \\ 0 & \text{if } A \notin H \end{cases}$$

We shall prove that λ^{μ} is a fuzzy HX subring of $\mathfrak{R}$.

Let $A, B \in \mathfrak{R}$.

i. Suppose $A, B \in H$, then $A - B \in H$ and $AB \in H$.

$\lambda^{\mu}(A) = t$, $\lambda^{\mu}(B) = t$, , $\lambda^{\mu}(A-B) = t$ and $\lambda^{\mu}(AB) = t$.

Hence , $\lambda^{\mu}(A-B) = t$
$$\geq \min\{t, t\}$$
$$\lambda^{\mu}(A-B) \geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\},$$
$$\lambda^{\mu}(AB) = t$$
$$\geq \min\{t, t\}$$
$$\lambda^{\mu}(AB) \geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\},$$

ii. Suppose $A \in H$ and $B \notin H$, then $A-B \notin H$ and $AB \notin H$.

$\lambda^{\mu}(A) = t$, $\lambda^{\mu}(B) = 0$, $\lambda^{\mu}(A-B) = 0$ and $\lambda^{\mu}(AB) = 0$.

Hence , $\lambda^{\mu}(A-B) = 0$

$\geq$ min { t , 0 }

$\lambda^{\mu}(A-B) \geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$,

$\lambda^{\mu}(AB) = 0$

$\geq$ min { t , 0}

$\lambda^{\mu}(AB) \geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$

iii. Suppose A, B $\notin$ H, then A–B $\notin$ H or A–B $\in$ H and AB $\notin$ H or AB $\in$ H

$\lambda^{\mu}(A) = 0$, $\lambda^{\mu}(B) = 0$, $\lambda^{\mu}(A-B)$ = t or 0, $\lambda^{\mu}(AB)$ = t or 0

Hence, $\lambda^{\mu}(A-B) \geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$,

$\lambda^{\mu}(AB) \geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$,

Thus in all cases, λ^{μ} is a fuzzy HX subring of $\mathfrak{R}$.

For this t $\in$ (0,1] , $U(\lambda^{\mu} ; t) = H$.

4.7 Remark

As a consequence of the **Theorem 4.5 and 4.6**, the level sub HX ring of a fuzzy HX subring λ^{μ} of a HX ring $\mathfrak{R}$ form a chain. Since $\lambda^{\mu}(Q) \geq \lambda^{\mu}(A)$ for all A in $\mathfrak{R}$ and therefore $U(\lambda^{\mu} ; t_0)$, where $\lambda^{\mu}(Q) = t_0$ is the smallest and we have the chain :

$\{Q\} = U(\lambda^{\mu} ; t_0) \subset U(\lambda^{\mu} ; t_1) \subset U(\lambda^{\mu} ; t_2) \subset \ldots \subset U(\lambda^{\mu} ; t_n) = \mathfrak{R}$,

where $t_0 > t_1 > t_2 > \ldots\ldots > t_n$, where, $t_0, t_1, t_2, \ldots\ldots, t_n \in [0, 1]$.

5. HOMOMORPHISM AND ANTI HOMOMORPHISM OF LEVEL SUBSETS OF FUZZY HX SUBRING

In this section, we introduce the concept of homomorphism and anti homomorphism of level subsets of a fuzzy HX subring of a HX ring and discuss some of its properties. Throughout this section, t $\in$ [0,1].

5.1 Theorem

Let $\mathfrak{R}_1$ and $\mathfrak{R}_2$ be any two HX rings. f: $\mathfrak{R}_1 \to \mathfrak{R}_2$ be an onto mapping and $U(\lambda^{\mu} ; t)$ be a level sub HX ring of a fuzzy HX subring of $\mathfrak{R}_1$. Then $U(f(\lambda^{\mu}) ; t) = f(U(\lambda^{\mu} ; t))$.

Proof

Let $\mathfrak{R}_1$ and $\mathfrak{R}_2$ be any two HX rings.

Let f : $\mathfrak{R}_1 \to \mathfrak{R}_2$ be a mapping.

Let $U(\lambda^{\mu} ; t)$ be a level sub HX ring of a fuzzy HX subring of $\mathfrak{R}_1$.

Let X $\in U(\lambda^{\mu} ; t)$ such that $f(X) \in U(f(\lambda^{\mu}) ; t)$,where t $\in$ [0,1] .

Let $f(X) \in U(f(\lambda^{\mu}); t)$ $\Leftrightarrow$ $(f(\lambda^{\mu}))\, f(X) \geq t$

$\Leftrightarrow \lambda^{\mu}(X) \geq t$

$\Leftrightarrow X \in U(\lambda^{\mu} ; t)$

$\Leftrightarrow f(X) \in f(U(\lambda^{\mu} ; t))$

Hence, $U(f(\lambda^{\mu}); t) = f(U(\lambda^{\mu} ; t))$.

5.2 Theorem

Let $\mathfrak{R}_1$ and $\mathfrak{R}_2$ be any two HX rings. Let f : $\mathfrak{R}_1 \to \mathfrak{R}_2$ be a mapping and $U(\eta^{\alpha}; t)$ be a level sub HX ring of a fuzzy HX subring of $\mathfrak{R}_2$. Then $U((f^{-1}(\eta^{\alpha}) ; t) = f^{-1}(U(\eta^{\alpha} ; t))$.

Proof

Let $\Re_1$ and $\Re_2$ be any two HX rings.

Let $f : \Re_1 \rightarrow \Re_2$ be a mapping.

Let $U(\eta^{\alpha} ;\ t)$ be a level sub HX ring of a fuzzy HX subring of $\Re_2$.

Let $X \in U(f^{-1}(\eta^{\alpha}) ; t) \Leftrightarrow (f^{-1}(\eta^{\alpha}))(X) \geq t$

$\Leftrightarrow \eta^{\alpha}(f(X)) \geq t$

$\Leftrightarrow f(X) \in U(\eta^{\alpha} ;\ t)$

$\Leftrightarrow X \in f^{-1}(U(\eta^{\alpha} ;\ t))$

Hence, $U(f^{-1}(\eta^{\alpha}) ; t) = f^{-1}(U(\eta^{\alpha} ;\ t))$.

5.3 Theorem

Let R_1 and R_2 be any two rings , $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let λ^{μ} be a fuzzy HX subring on $\Re_1$. If f: $\Re_1 \rightarrow \Re_2$ is a homomorphism and onto, then the image of a level sub HX ring $U(\lambda^{\mu} ;\ t)$ of a fuzzy HX subring λ^{μ} of a HX ring $\Re_1$ is a level sub HX ring $U(f(\lambda^{\mu});\ t)$ of a fuzzy HX subring $f(\lambda^{\mu})$ of a HX ring $\Re_2$.

Proof

Let R_1 and R_2 be any two rings and $f : \Re_1 \rightarrow \Re_2$ be a homomorphism.

Let λ^{μ} be a fuzzy HX subring of a HX ring $\Re_1$. Clearly, $f(\lambda^{\mu})$ is a fuzzy HX subring of a HX ring $\Re_2$. Let X and Y in $\Re_1$, implies f (X) and f (Y) in $\Re_2$.

Let $U(\lambda^{\mu} ;\ t)$ is a level sub HX ring of a fuzzy HX subring λ^{μ} of a HX ring $\Re_1$.

Choose t in such a way that $X, Y \in U(\lambda^{\mu} ;\ t)$ and hence $X-Y$, $XY \in U(\lambda^{\mu} ;\ t)$.

Then, $\lambda^{\mu}(X) \geq t$ and $\lambda^{\mu}(Y) \geq t$.

Also $\lambda^{\mu}(X-Y) \geq t$, $\lambda^{\mu}(XY) \geq t$,

We have to prove that $U(f(\lambda^{\mu}) ;\ t)$ is a level sub HX ring of a fuzzy HX subring $f(\lambda^{\mu})$ of a HX ring $\Re_2$.

Now, Let $f(X) , f(Y) \in U(f(\lambda^{\mu});\ t)$.

$(f(\lambda^{\mu}))(f(X)) = \lambda^{\mu}(X) \geq t$, implies that $(f(\lambda^{\mu}))(f(X)) \geq t$

$(f(\lambda^{\mu}))(f(Y)) = \lambda^{\mu}(Y) \geq t$, implies that $(f(\lambda^{\mu}))(f(Y)) \geq t$.

i. $(f(\lambda^{\mu}))(f(X) - f(Y)) = (f(\lambda^{\mu}))(f(X-Y))$,

$= \lambda^{\mu}(X-Y)$

$(f(\lambda^{\mu}))(f(X) - f(Y)) \geq t$

$(f(\lambda^{\mu}))(f(X) - f(Y)) \geq t$.

$(f(X) - f(Y)) \in U(f(\lambda^{\mu});\ t)$.

ii. $(f(\lambda^{\mu}))(f(X)\, f(Y)) = (f(\lambda^{\mu}))(f(XY))$,

$= \lambda^{\mu}(XY)$

$\geq t$

$(f(\lambda^{\mu}))(f(X)\, f(Y)) \geq t$.

$(f(X)\, f(Y)) \in U(f(\lambda^{\mu});\ t)$.

Hence, $U(f(\lambda^{\mu}) ; t)$ is a level sub HX ring of a fuzzy HX subring $f(\lambda^{\mu})$ of a HX ring $\Re_2$.

5.4 Theorem

Let R_1 and R_2 be any two rings , $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let η^{α} be a fuzzy HX subring on $\Re_2$. If f: $\Re_1 \rightarrow \Re_2$ is a homomorphism on HX rings. Let $U(\eta^{\alpha} ;\ t)$ be a level sub HX ring of a fuzzy HX subring η^{α} of a HX ring $\Re_2$ then $U(f^{-1}(\eta^{\alpha});\ t)$ is a level sub HX ring of a fuzzy HX subring $f^{-1}(\eta^{\alpha})$ of a HX ring $\Re_1$.

Proof

Let R_1 and R_2 be any two rings and $f : \Re_1 \rightarrow \Re_2$ be a homomorphism.

Let η^α be a fuzzy HX subring of a HX ring $\Re_2$. Clearly, $f^{-1}(\eta^\alpha)$ is a fuzzy HX subring of a HX ring $\Re_1$. Let X and Y in $\Re_1$, implies f(X) and f(Y) in $\Re_2$.

Let $U(\eta^\alpha;\ t)$ be a level sub HX ring of a fuzzy HX subring η^α of a HX ring $\Re_2$.

Choose t in such a way that $X, Y \in U(\eta^\alpha;\ t)$ and hence, $X - Y$, $XY \in U(\eta^\alpha;\ t)$.

Then, $\eta^\alpha(f(X)) \geq t$ and $\eta^\alpha(f(Y)) \geq t$.

Also, $\eta^\alpha(f(X)-f(Y)) \geq t$, $\eta^\alpha(f(X)f(Y)) \geq t$,

We have to prove that $U(f^{-1}(\eta^\alpha);\ t)$ is a level sub HX ring of a fuzzy HX subring $f^{-1}(\eta^\alpha)$ of a HX ring $\Re_1$.

Now, Let $X, Y \in U(f^{-1}(\eta^\alpha);\ t)$.

$(f^{-1}(\eta^\alpha))(X) = \eta^\alpha(f(X)) \geq t$, implies that $(f^{-1}(\eta^\alpha))(X) \geq t$

$(f^{-1}(\eta^\alpha))(Y) = \eta^\alpha(f(Y)) \geq t$, implies that $(f^{-1}(\eta^\alpha))(Y) \geq t$

i. $(f^{-1}(\eta^\alpha))(X-Y) = \eta^\alpha(f(X-Y))$
$= \eta^\alpha(f(X) - f(Y))$
$\geq t$
$(f^{-1}(\eta^\alpha))(X-Y) \geq t$
$X-Y \in U(f^{-1}(\eta^\alpha);\ t)$.

ii. $(f^{-1}(\eta^\alpha))(XY) = \eta^\alpha(f(XY))$
$= \eta^\alpha(f(X)f(Y))$,
$\geq t$
$(f^{-1}(\eta^\alpha))(XY) \geq t$
$XY \in U(f^{-1}(\eta^\alpha);\ t)$.

Hence, $U(f^{-1}(\eta^\alpha);\ t)$ is a level sub HX ring of a fuzzy HX subring of a HX ring $\Re_1$.

5.5 Theorem

Let R_1 and R_2 be any two rings, $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let λ^μ be a fuzzy HX subring on $\Re_1$. If $f : \Re_1 \rightarrow \Re_2$ is an anti homomorphism and onto, then the image of a level sub HX ring $U(\lambda^\mu;\ t)$ of a fuzzy HX subring λ^μ of a HX ring $\Re_1$ is a level sub HX ring $U(f(\lambda^\mu);t)$ of a fuzzy HX subring $f(\lambda^\mu)$ of a HX ring $\Re_2$.

Proof

Let R_1 and R_2 be any two rings and $f : \Re_1 \rightarrow \Re_2$ be an anti homomorphism.

Let λ^μ be a fuzzy HX subring of a HX ring $\Re_1$. Clearly, $f(\lambda^\mu)$ is a fuzzy HX subring of a HX ring $\Re_2$. Let X and Y in $\Re_1$, implies f(X) and f(Y) in $\Re_2$.

Let $U(\lambda^\mu;\ t)$ is a level sub HX ring of a fuzzy HX subring λ^μ of a HX ring $\Re_1$.

Choose t in such a way that $X, Y \in U(\lambda^\mu;\ t)$ and hence $X-Y, XY \in U(\lambda^\mu;\ t)$.

Then, $\lambda^\mu(X) \geq t$ and $\lambda^\mu(Y) \geq t$.

Also $\lambda^\mu(Y - X) \geq t$, $\lambda^\mu(YX) \geq t$,

We have to prove that $U(f(\lambda^\mu);\ t)$ is a level sub HX ring of a fuzzy HX subring $f(\lambda^\mu)$ of a HX ring $\Re_2$.

Now, Let $f(X), f(Y) \in U(f(\lambda^\mu);\ t)$.

$(f(\lambda^\mu))(f(X)) = \lambda^\mu(X) \geq t$, implies that $(f(\lambda^\mu))(f(X)) \geq t$

$(f(\lambda^\mu))(f(Y)) = \lambda^\mu(Y) \geq t$, implies that $(f(\lambda^\mu))(f(Y)) \geq t$.

i. $(f(\lambda^\mu))(f(X)-f(Y)) = (f(\lambda^\mu))(f(Y-X))$,
$= \lambda^\mu(Y-X)$

$\geq t$

$(f(\lambda^{\mu}))(f(X)-f(Y)) \geq t$.

$(f(X)-f(Y)) \in U(f(\lambda^{\mu});\ t)$.

ii. $(f(\lambda^{\mu}))(f(X)\,f(Y)) = (f(\lambda^{\mu}))(f(YX))$,

$= \lambda^{\mu}(YX)$

$\geq t$

$(f(\lambda^{\mu}))(f(X)f(Y)) \geq t$.

$(f(X)\,(f(Y)) \in U(f(\lambda^{\mu});\ t)$.

Hence, $U(f(\lambda^{\mu})\ ;\ t)$ is a level sub HX ring of a fuzzy HX subring $f(\lambda^{\mu})$ of a HX ring $\Re_2$.

5.6 Theorem

Let R_1 and R_2 be any two rings and $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let η^{α} be a fuzzy HX subring on $\Re_2$. If f: $\Re_1 \rightarrow \Re_2$ is an anti homomorphism onto HX rings. Let $U(\eta^{\alpha}\ ;\ t)$ be a level sub HX ring of a fuzzy HX subring η^{α} of a HX ring $\Re_2$ then $U(f^{-1}(\eta^{\alpha});\ t)$ is a level sub HX ring of a fuzzy HX subring $f^{-1}(\eta^{\alpha})$ of a HX ring $\Re_1$.

Proof

Let R_1 and R_2 be any two rings and f: $\Re_1 \rightarrow \Re_2$ be an anti homomorphism.

Let η^{α} be a fuzzy HX subring of a HX ring $\Re_2$. Clearly, $f^{-1}(\eta^{\alpha})$ is a fuzzy HX subring of a HX ring $\Re_1$. Let X and Y in $\Re_1$, implies f(X) and f(Y) in $\Re_2$.

Let $U(\eta^{\alpha};\ t)$ be a level sub HX ring of a fuzzy HX subring η^{α} of a HX ring $\Re_2$.

Choose t in such a way that $X, Y \in U(\eta^{\alpha};\ t)$ and hence, $X-Y,\ XY \in U(\eta^{\alpha}\ ;\ t)$.

Then, $\eta^{\alpha}(f(X)) \geq t$ and $\eta^{\alpha}(f(Y)) \geq t$.

Also $\eta^{\alpha}(f(Y)-f(X)) \geq t$, $\eta^{\alpha}(f(Y)f(X)) \geq t$,

We have to prove that $U(f^{-1}(\eta^{\alpha});\ t)$ is a level sub HX ring of a fuzzy HX subring $f^{-1}(\eta^{\alpha})$ of a HX ring $\Re_1$.

Now, Let $X, Y \in U(f^{-1}(\eta^{\alpha});\ t)$.

$(f^{-1}(\eta^{\alpha}))(X) = \eta^{\alpha}(f(X)) \geq t$, implies that $(f^{-1}(\eta^{\alpha}))(X) \geq t$

$(f^{-1}(\eta^{\alpha}))(Y) = \eta^{\alpha}(f(Y)) \geq t$, implies that $(f^{-1}(\eta^{\alpha}))(Y) \geq t$.

i. $(f^{-1}(\eta^{\alpha}))(X-Y) = \eta^{\alpha}(f(X-Y))$

$= \eta^{\alpha}[f(Y)-f(X)]$,

$\geq t$

$(f^{-1}(\eta^{\alpha}))(X-Y) \geq t$

$X-Y \in U(f^{-1}(\eta^{\alpha});\ t)$.

ii. $(f^{-1}(\eta^{\alpha}))(XY) = \eta^{\alpha}(f(YX))$

$= \eta^{\alpha}(f(Y)f(X))$,

$\geq t$

$(f^{-1}(\eta^{\alpha}))(XY) \geq t$

$XY \in U(f^{-1}(\eta^{\alpha});\ t)$.

Hence, $U(f^{-1}(\eta^{\alpha})\ ;\ t)$ is a level sub HX ring of a fuzzy HX subring $f^{-1}(\eta^{\alpha})$ of a HX ring $\Re_1$.

REFERENCES

1. Bing-xueYao and Yubin-Zhong, The construction of power ring, Fuzzy information and Engineering (ICFIE),ASC 40,pp 181-187 (2007).
2. Bing-xueYao and Yubin-Zhong, Upgrade of algebraic structure of ring, Fuzzy information and Engineering 2:pp 219-228(2009).
3. Li Hong Xing, HX group, BUSEFAL,33(4), pp 31-37 (1987).
4. Liu. W.J., Fuzzy invariant subgroups and fuzzy ideals, Fuzzy sets and systems, 8:pp 133-139.
5. Li Hong Xing, HX ring, BUSEFAL ,34(1),pp. 3-8 (1988).
6. Muthuraj.R, Ramila Gandhi.N, Homomorphism and anti homomorphism of fuzzy HX ideals of a HX ring, Discovery, Volume 21, Number 65, pp 20-24(2014).
7. Palaniappan. N & Muthuraj.R, The homomorphism, anti-homomorphism of a fuzzy and an anti-fuzzy groups, Varahmihir Journal of Mathematical Sciences, Vol.4 No.2 387-399(2004).
8. Rosenfeld. A., Fuzzy groups,J.Math.Anal.,35 ,pp 512-517(1971).
9. Shaoquan Sun,Some properties of the direct product of fuzzy algebras, pp 1-5.
   Wang Qing –hua ,Fuzzy rings and fuzzy subrings , pp 1-6.
10. YAO Bing- xue, Homomorphism and Isomorphism of Regular Power Ring, Chinese quarterly
11. journal of mathematics,Vol. 15 No. 2 ,pp 23-28 (2000). Zadeh.L.A., Fuzzy sets,Information and control,8,pp 338-353.